Covariant Light-Front Approach for s-wave and p-wave Mesons

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We study the decay constants and form factors of the ground-state s-wave and low-lying p-wave mesons within a covariant light-front approach. Numerical results of the $B \to D^{**}$ transition form factors, where D^{**} denotes generically a p-wave charmed meson, are compared with other model calculations. Predictions on the decay rates for $\overline{B} \to D^{**}\pi$, $D^{**}\rho$, $\overline{D}_s^{**}D^{(*)}$ by using these decay constants and form factors are in agreement with the experimental data. The universal Isgur-Wise functions, ξ, $\tau_{1/2}$, $\tau_{3/2}$ are obtained and are used to test the Bjorken and Uraltsev sum rules.

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I. INTRODUCTION

Mesonic weak transition form factors and decay constants are two of the most important ingredients in the study of hadronic weak decays of mesons. There exist many different model calculations. The light-front quark model [1, 2] is the only relativistic quark model in which a consistent and fully relativistic treatment of quark spins and the center-of-mass motion can be carried out. This model is very suitable to study hadronic form factors. Especially, as the recoil momentum increases (corresponding to a decreasing q^2), we have to start considering relativistic effects seriously. In particular, at the maximum recoil point $q^2 = 0$ where the final-state meson could be highly relativistic, there is no reason to expect that the non-relativistic quark model is still applicable.

The relativistic quark model in the light-front approach has been employed to obtain decay constants and weak form factors [3, 4, 5, 6]. There exist, however, some ambiguities and even some inconsistencies in extracting the physical quantities. Well known examples are the vector decay constant f_V, the form factor $F_0(q^2)$ in the pseudoscalar to pseudoscalar transition [7] and the electromagnetic form factor $F_2(q^2)$ of the vector meson (see e.g. [8]). A covariant model has been constructed in [9] for heavy mesons within the framework of heavy quark effective theory and the results are free from above mentioned ambiguities.

Without appealing to the heavy quark limit, a covariant approach of the light-front model for the usual pseudoscalar and vector mesons has been put forward by Jaus [7]. The procedure of calculation can be separated into four steps: (a) The starting point of the covariant approach is to consider the corresponding covariant Feynman amplitudes in meson transitions as shown in Fig. 1. (b) One can pass to the light-front approach by using the light-front decomposition of the internal momentum in covariant Feynman momentum loop integrals and integrating out the $p^- = p^0 - p^3$ component [10]. (c) At this stage one can then apply some widely-used vertex functions in the conventional light-front approach after p^- integration. It is pointed out by Jaus that in going from the manifestly covariant Feynman integral to the light-front one, the latter is no longer covariant as it receives additional spurious contributions proportional to the lightlike vector $\tilde{\omega}^\mu = (1, 0, 0, -1)$. (d) This spurious contribution is cancelled after correctly performing the integration, namely, by the inclusion of the zero mode contribution [11], so that the result is guaranteed to be covariant. It should be noted that in [7] a simple covariant power-law-like vertex function is used in step (a). Once a covariant vertex function is used in step (a), the above procedure should give identical results to the direct integration of Feynman integration via the usual technique. The power-law-like vertex function does not lead to a satisfactory phenomenological result when comparing to some widely used Gaussian form vertex functions in step (c). Since the covariant counterpart of the Gaussian like vertex function is not explicitly known, a use of it in step (c) may lead to some residue spurious contributions. These corrections to decay constants and form factors are worked out in [12]. We check that these corrections are small in the decay constant (within 10%) and form factors (within 2%). In [13], we have extended the covariant analysis of the light-front model in [7] to even-parity, p-wave mesons. Since the residue spurious corrections have not been worked out in the p-wave meson case, we shall follow the procedure of [7] for the extension.

Interest in even-parity charmed mesons has been revived by recent discoveries of two narrow resonances: the 0^+ state $D^*_{s0}(2317)$ [14] and the $P_1^{1/2}$ state $D_{s1}(2460)$ [15], and two broad resonances, $D^*_0(2308)$ and $D_1(2427)$ [16]. Furthermore, the hadronic B decays such as $B \to D^{**}\pi$ and $B \to D_s^{**}\overline{D}$ have been recently observed, where D^{**} denotes a p-wave charmed meson. A theoretical study of them requires the information of the $B \to D^{**}$ form factors and the decay constants of D^{**} and D_s^{**}. In the meantime, three body decays

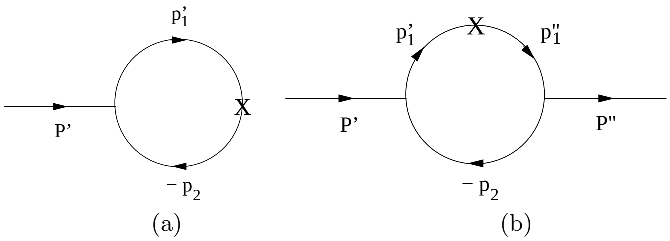

FIG. 1: Feynman diagrams for (a) meson decay and (b) meson transition amplitudes, where $P'^{(\prime\prime)}$ is the incoming (outgoing) meson momentum, $p_1'^{(\prime\prime)}$ is the quark momentum, p_2 is the anti-quark momentum and X denotes the corresponding $V-A$ current vertex.

of B mesons have been recently studied at the B factories: BaBar and Belle. The Dalitz plot analysis allows one to see the structure of exclusive quasi-two-body intermediate states in the three-body signals. The p-wave resonances observed in three-body decays begin to emerge. Theoretically, the Isgur-Scora-Grinstein-Wise (ISGW) quark model [17] is so far the only model in the literature that can provide a systematical estimate of the transition of a ground-state s-wave meson to a low-lying p-wave meson. However, this model and, in fact, many other models in $P \to P, V$ (P: pseudoscalar meson, V: vector meson) calculations, are based on the non-relativistic constituent quark picture. As noted in passing, the final-state meson at the maximum recoil point $q^2 = 0$ or in heavy-to-light transitions could be highly relativistic. It is thus important to consider a relativistic approach.

There are some theoretical constraints implied by heavy quark symmetry (HQS) in the case of heavy-to-heavy transitions and heavy-to-vacuum decays [18]. It is important to check if the calculated form factors and decay constants do satisfy these constraints. Furthermore, under HQS the number of the independent form factors is reduced and they are related to some universal Isgur-Wise (IW) functions. The relevant IW functions, namely, ξ, $\tau_{1/2}$ and $\tau_{3/2}$ are obtained. One can then study some properties of these IW functions, including the slopes and sum rules [19, 20].

II. DECAY CONSTANT

The decay constants for $J = 0, 1$ mesons are defined by the matrix elements

$$\begin{aligned}
\langle 0|A_\mu|P(P')\rangle &= i f_P P'_\mu, \qquad \langle 0|V_\mu|S(P')\rangle = f_S P'_\mu,\\
\langle 0|V_\mu|V(P',\varepsilon')\rangle &= M'_V f_V \varepsilon'_\mu,\\
\langle 0|A_\mu|\,{}^{3(1)}A(P',\varepsilon')\rangle &= M'_{{}^3A({}^1A)} f_{{}^3A({}^1A)} \varepsilon'_\mu,
\end{aligned}$$

where the ${}^{2S+1}L_J = {}^1S_0$, ${}^3P_0$, ${}^3S_1$, ${}^3P_1$, ${}^1P_1$ and ${}^3P_2$ states of $q_1'\bar q_2$ mesons are denoted by P, S, V, ${}^3A$, ${}^1A$ and T, respectively. Note that a ${}^3P_2$ state cannot be produced by a current. It is useful to note that in the SU(N)-flavor limit ($m_1' = m_2$) we should have vanishing f_S and $f_{{}^1A}$ [21]. These can be easily seen from charge conjugation. Under charge conjugation $V_\mu \to -V_\mu$, while $A_\mu \to A_\mu$. For a charge conjugated state, we have $C = (-)^{L+S}$. Thus, we must have $f_S = f_{{}^1A} = 0$ for these charge conjugated states. Through SU(N) symmetry the above constraint should apply to all other states in the same multiples.

Furthermore, in the heavy quark limit ($m_1' \to \infty$), the heavy quark spin s_Q decouples from the other degrees of freedom so that s_Q and the total angular momentum of the light antiquark j are separately good quantum numbers. Hence, it is more convenient to use the $L_J^j = P_2^{3/2}$, $P_1^{3/2}$, $P_1^{1/2}$ and $P_0^{1/2}$ basis. It is obvious that the first and the last of these states are ${}^3P_2$ and ${}^3P_0$, respectively, while [22]

$$\begin{aligned}
\left|P_1^{3/2}\right\rangle &= \sqrt{\frac{2}{3}}\,\left|{}^1P_1\right\rangle + \frac{1}{\sqrt{3}}\,\left|{}^3P_1\right\rangle,\\
\left|P_1^{1/2}\right\rangle &= \frac{1}{\sqrt{3}}\,\left|{}^1P_1\right\rangle - \sqrt{\frac{2}{3}}\,\left|{}^3P_1\right\rangle. \qquad (1)
\end{aligned}$$

Since, decay constants should be identical within each multiplet, ($S_0^{1/2}$, $S_1^{1/2}$), ($P_0^{1/2}$, $P_1^{1/2}$), ($P_1^{3/2}$, $P_2^{3/2}$), heavy quark symmetry (HQS) requires [18, 23]

$$f_V = f_P, \qquad f_{A^{1/2}} = f_S, \qquad f_{A^{3/2}} = 0, \qquad (2)$$

where we have denoted the $P_1^{1/2}$ and $P_1^{3/2}$ states by $A^{1/2}$ and $A^{3/2}$, respectively. It is important to check if the calculated decay constants satisfy the non-trivial SU(N)-flavor and HQS relations.

We now follow [7] to evaluate meson decay constants and obtain

$$\begin{aligned}
f_P &= \frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{4h'_P}{x_1x_2(M'^2-M_0'^2)}(m_1'x_2+m_2x_1),\\
f_S &= \frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{4h'_S}{x_1x_2(M'^2-M_0'^2)}(m_1'x_2-m_2x_1),\\
f_V &= \frac{N_c}{4\pi^3}\int dx_2 d^2p'_\perp \frac{h'_V}{x_1x_2(M'^2-M_0'^2)}\\
&\quad\times\left[x_1M_0'^2 - m_1'(m_1'-m_2) - p_\perp'^2 + \frac{m_1'+m_2}{w'_V}\,p_\perp'^2\right],\\
f_{{}^3A} &= -\frac{N_c}{4\pi^3}\int dx_2 d^2p'_\perp \frac{h'_{{}^3A}}{x_1x_2(M'^2-M_0'^2)}\\
&\quad\times\left[x_1M_0'^2 - m_1'(m_1'+m_2) - p_\perp'^2 - \frac{m_1'-m_2}{w'_{{}^3A}}\,p_\perp'^2\right],\\
f_{{}^1A} &= \frac{N_c}{4\pi^3}\int dx_2 d^2p'_\perp \frac{h'_{{}^1A}}{x_1x_2(M'^2-M_0'^2)}\left(\frac{m_1'-m_2}{w'_{{}^1A}}\,p_\perp'^2\right),
\end{aligned} \qquad (3)$$

TABLE I: The input parameter β (in units of GeV) in the Gaussian-type wave function.

$^{2S+1}L_J$	$\beta_{u\bar{d}}$	$\beta_{s\bar{u}}$	$\beta_{c\bar{u}}$	$\beta_{c\bar{s}}$	$\beta_{b\bar{u}}$
$^1S_0$	0.3102	0.3864	0.4496	0.4945	0.5329
$^3P_0$	β_{a_1}	$\beta_{K(^3P_1)}$	0.3305	0.3376	0.4253
$^3S_1$	0.2632	0.2727	0.3814	0.3932	0.4764
$^3P_1$	0.2983	0.303	0.3305	0.3376	0.4253
$^1P_1$	β_{a_1}	$\beta_{K(^3P_1)}$	0.3305	0.3376	0.4253

TABLE II: Mesonic decay constants (in units of MeV) obtained by using Eq. (3). Those in parentheses are taken as inputs to determine the corresponding β's shown in Table I.

$^{2S+1}L_J$	$f_{u\bar{d}}$	$f_{s\bar{u}}$	$f_{c\bar{u}}$	$f_{c\bar{s}}$	$f_{b\bar{u}}$
$^1S_0$	(131)	(160)	(200)	(230)	(180)
$^3P_0$	0	22	86	71	112
$^3S_1$	(216)	(210)	(220)	(230)	(180)
$^3P_1$	(-203)	-186	-127	-121	-123
$^1P_1$	0	11	45	38	68
$P_1^{1/2}$	–	–	130	122	140
$P_1^{3/2}$	–	–	-36	-38	-15

where M' are meson masses, m_1', m_2 are quark masses, h' are vertex functions, M_0' are kinetic masses and x_i are momentum fractions. Since h' and M_0' are symmetric under the exchange of 1 and 2 in the $m_1' = m_2$ limit. It is clear that $f_S = f_{^1A} = 0$ for $m_1' = m_2$. The SU(N)-flavor constraints on f_S and $f_{^1A}$ are thus satisfied.

In order to have numerical results for decay constants, we need to specify the constituent quark masses and the parameter β appearing in the Gaussian-type wave function. For constituent quark masses (in units of GeV) we use $m_{u,d} = 0.26$, $m_s = 0.37$, $m_c = 1.40$, $m_b = 4.64$. Note that m_s and m_c are constrained from the measured form-factor ratios in semileptonic $D \to K^* \ell \bar{\nu}$ decays [24]. Shown in Tables I and II are the input parameter β and decay constants, respectively. In Table II the decay constants in parentheses are used to determine β. For the purpose of an estimation, for p-wave mesons in D, D_s and B systems we shall use the β parameters in the ISGW2 model [25] up to some simple scaling. Two remarks are in order: (i) The values of the parameter β_V presented in Table I are slightly smaller than the ones obtained in the earlier literature. It is interesting that β_V in the ISGW2 model also have a similar reduction due to L-S interaction, which is neglected in the original ISGW model in the mass spectrum calculation. (ii) The β parameters for p-wave states of D, D_s and B systems are the smallest when compared to $\beta_{P,V}$.

It is clear that the decay constant of light scalar resonances is suppressed relative to that of the pseudoscalar mesons owing to the small mass difference between the constituent quark masses. However, as shown in Table II, the suppression becomes less effective for heavy scalar mesons because of heavy and light quark mass imbalance. The prediction of $f_S = 21$ MeV for the scalar meson in the $s\bar{u}$ content (see Table II) is most likely designated for the $K_0^*(1430)$ state. Notice that this prediction is slightly smaller than the result of 42 MeV obtained in [26] based on the finite-energy sum rules, and far less than the estimate of (70 ± 10) MeV in [27]. It is worth remarking that even if the light scalar mesons are made from 4 quarks, the decay constants of the neutral scalars $\sigma(600)$, $f_0(980)$ and $a_0^0(980)$ must vanish owing to charge conjugation invariance.

In principle, the decay constant of the scalar strange charmed meson D_{s0}^* can be determined from the hadronic decay $B \to \overline{D} D_{s0}^*$ since it proceeds only via external W-emission. Naively, it is expected that D_{s0}^* has a slightly smaller decay constant than that of D_0^*, in contrast to the pseudoscalar case where $f_{D_s} > f_D$. However, a recent measurement of the $D\bar{D}_{s0}^*$ production in B decays by Belle [28] seems to indicate a $f_{D_{s0}^*}$ of order 60 ± 10 MeV [29] which is close to the expectation of 71 MeV (see Table II). The smallness of this decay constant is due to the fact that comparing to the $c\bar{u}$ system, the $c\bar{s}$ system is closer to the SU(N) limit, where $f_S = 0$. We will return to B decays in the next section.

Except for a_1 and b_1 mesons which cannot have mixing because of the opposite C-parities, physical strange axial-vector mesons are the mixture of $^3P_1$ and $^1P_1$ states, while the heavy axial-vector resonances are the mixture of $P_1^{1/2}$ and $P_1^{3/2}$. For example, $K_1(1270)$ and $K_1(1400)$ are the mixture of $K_{^3P_1}$ and $K_{^1P_1}$ (denoted by K_{1A} and K_{1B}, respectively, by PDG [30]) owing to the mass difference of the strange and non-strange light quarks:

$$
\begin{aligned}
K_1(1270) &= K_{^3P_1} \sin\theta + K_{^1P_1} \cos\theta, \\
K_1(1400) &= K_{^3P_1} \cos\theta - K_{^1P_1} \sin\theta,
\end{aligned}
\tag{4}
$$

with $\theta \approx -58^\circ$ as implied from the study of $D \to K_1(1270)\pi$, $K_1(1400)\pi$ decays [31]. We use $f_{K_1(1270)} = 175$ MeV [31] to fix $\beta_{K(^3P_1)} \simeq \beta_{K(^1P_1)} = 0.303$ GeV and obtain $f_{K_1(1400)} = -87$ MeV. Note that these $\beta_{K(^3P_1)}$, $\beta_{K(^1P_1)}$ are close to β_{K^*}.

For D, D_s and B systems, it is clear from Table II that $|f_{A^{3/2}}| \ll f_S \lesssim f_{A^{1/2}}$, in accordance with the expectation from HQS [cf. Eq. (2)]. In fact, the HQS relations on decay constants Eq. (2) are verified in the HQ limit [13].

III. FORM FACTORS

Form factors for $P \to P, V$ and P to low-lying p-wave meson transitions are defined by [17, 32]

$$
\begin{aligned}
\langle P(P'')|V_\mu|P(P')\rangle &= \left(P_\mu - \frac{M'^2 - M''^2}{q^2}\, q_\mu\right) F_1^{PP}(q^2) + \frac{M'^2 - M''^2}{q^2}\, q_\mu\, F_0^{PP}(q^2), \\
\langle V(P'',\varepsilon'')|V_\mu|P(P')\rangle &= -\frac{1}{M'+M''}\, \epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu} P^\alpha q^\beta V^{PV}(q^2), \\
\langle V(P'',\varepsilon'')|A_\mu|P(P')\rangle &= i\Big\{(M'+M'')\varepsilon''^*_\mu A_1^{PV}(q^2) - \frac{\varepsilon''^*\cdot P}{M'+M''}\, P_\mu A_2^{PV}(q^2) - 2M''\,\frac{\varepsilon''^*\cdot P}{q^2}\, q_\mu \big[A_3^{PV}(q^2) - A_0^{PV}(q^2)\big]\Big\}, \\
\langle S(P'')|A_\mu|P(P')\rangle &= -i\left[\left(P_\mu - \frac{M'^2 - M''^2}{q^2}\, q_\mu\right) F_1^{PS}(q^2) + \frac{M'^2 - M''^2}{q^2}\, q_\mu\, F_0^{PS}(q^2)\right], \\
\langle A(P'',\varepsilon'')|V_\mu|P(P')\rangle &= -i\left\{(m_P - m_A)\varepsilon^*_\mu V_1^{PA}(q^2) - \frac{\varepsilon^*\cdot P'}{m_P - m_A} P_\mu V_2^{PA}(q^2) - 2m_A \frac{\varepsilon^*\cdot P'}{q^2} q_\mu \left[V_3^{PA}(q^2) - V_0^{PA}(q^2)\right]\right\}, \\
\langle A(P'',\varepsilon'')|A_\mu|P(P')\rangle &= -\frac{1}{m_P - m_A}\, \epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu} P^\rho q^\sigma A^{PA}(q^2), \\
\langle T(P'',\varepsilon'')|V_\mu|P(P')\rangle &= h(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu\lambda} P_\lambda P^\alpha q^\beta, \\
\langle T(P'',\varepsilon'')|A_\mu|P(P')\rangle &= -i\Big\{k(q^2)\varepsilon''^*_{\mu\nu} P^\nu + \varepsilon''^*_{\alpha\beta} P^\alpha P^\beta [P_\mu b_+(q^2) + q_\mu b_-(q^2)]\Big\}.
\end{aligned} \tag{5}
$$

with $P = P' + p''$, $q = p' - p''$, $F_1^{PP}(0) = F_0^{PP}(0)$, $A_3^{PV}(0) = A_0^{PV}(0)$, $V_3^{PA}(0) = V_0^{PA}(0)$, where

$$
\begin{aligned}
A_3^{PV}(q^2) &= \frac{M'+M''}{2M''}\, A_1^{PV}(q^2) - \frac{M'-M''}{2M''}\, A_2^{PV}(q^2), \\
V_3^{PA}(q^2) &= \frac{m_P - m_A}{2m_A}\, V_1^{PA}(q^2) - \frac{m_P + m_A}{2m_A}\, V_2^{PA}(q^2).
\end{aligned} \tag{6}
$$

The definition here for dimensionless $P \to A$ transition form factors differs than that in [29] where the coefficients $(m_P \pm m_A)$ are replaced by $(m_P \mp m_A)$.

We follow [7] to obtain $P \to P, V$ form factors and extend the formalism to the p-wave meson case. The calculation is done in the $q^+ = 0$ frame, where $q^2 \leq 0$. We follow [5] to analytically continue form factors to timelike region.

To proceed we find that the momentum dependence of form factors in the spacelike region can be well parameterized and reproduced in the three-parameter form:

$$
F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2} \tag{7}
$$

$$
F(q^2) = \frac{F(0)/(1 - q^2/m_B^2)}{[1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2]}, \tag{8}
$$

for $B \to M$ transitions where the latter one is for the form factor V_2 and the former one is for the rest. The parameters a, b and $F(0)$ are first determined in the spacelike region. We then employ this parametrization to determine the physical form factors at $q^2 \geq 0$.

In Table III we show the form factors and their q^2 dependence for the B to D, D^*, $D_0^*(2308)$, $D_1^{1/2}$, $D_1^{3/2}$, $D_2^*(2460)$ transitions. Other results, including $B(D)$ to π, ρ, $a_0(1450)$, $a_1(1260)$, $b_1(1235)$, $a_2(1320)$, K, K^*, $K_0^*(1430)$, $K_{^1P_1}$, $K_{^3P_1}$, $K_2^*(1430)$ transition form factors can be found in [13].

In Table IV, decay rates for $\overline{B} \to D^{**}\pi$, $D^{**}\rho$, $\overline{D}_s^{**} D^{(*)}$ obtained in light-front and ISGW2 models, respectively, are given. These are updated from the analysis of [29] by using the decay constants and form factors shown in Tables II and III, respectively [33]. Since decay constants for p-wave mesons are not provided in ISGW2 model, our decay constants and their form factors are used for the ISGW2 results quoted.

Several remarks are in order:

1. For heavy-to-heavy transitions such as $B \to D$, D^*, D^{**}, the sign of various form factors can be checked by heavy quark symmetry. Our results are indeed in accordance with HQS.

2. It is pointed out in [6] that for $B \to D, D^*$ transitions, the form factors F_1, A_0, A_2, V exhibit a dipole behavior, while F_0 and A_1 show a monopole dependence. An inspection of Table III indicates that form factors F_0^{BD} and $A_1^{BD^*}$ have a monopole behavior, while F_1^{BD}, V^{BD^*} and $A^{BD_1^{3/2}}$ have a dipole dependence.

3. Our numerical result for k is too sensitive to the β parameter. The k shown in Table III are determined from h and $b_+ - b_-$ through HQ relations instead.

4. We see from the comparison of LF and ISGW2 results in Table III that the form factors at small q^2 obtained in the covariant light-front and ISGW2 models differ not more than 40%. Relativistic effects are mild in $B \to D$ transition, but they could

TABLE III: Form factors for $B \to D,\ D^*, D_0^*, D_1^{1/2}, D_1^{3/2}, D_2^*$ transitions are fitted to the 3-parameter form Eq. (7) except for the form factor V_2 denoted by * for which the fit formula Eq. (8) is used. For the purpose of comparison we quote the result of ISGW2 in the lower half table.

F	$F(0)$	a	b	F	$F(0)$	a	b
F_1^{BD}	0.67	1.25	0.39	F_0^{BD}	0.67	0.65	0.00
V^{BD^*}	0.75	1.29	0.45	$A_0^{BD^*}$	0.64	1.30	0.31
$A_1^{BD^*}$	0.63	0.65	0.02	$A_2^{BD^*}$	0.61	1.14	0.52
$F_1^{BD_0^*}$	0.24	1.03	0.27	$F_0^{BD_0^*}$	0.24	-0.49	0.35
$A^{BD_1^{1/2}}$	-0.12	0.71	0.18	$V_0^{BD_1^{1/2}}$	0.08	1.28	-0.29
$V_1^{BD_1^{1/2}}$	-0.19	-1.25	0.97	$V_2^{BD_1^{1/2}}$	-0.12	0.67	0.20
$A^{BD_1^{3/2}}$	0.23	1.17	0.39	$V_0^{BD_1^{3/2}}$	0.47	1.17	0.03
$V_1^{BD_1^{3/2}}$	0.55	-0.19	0.27	$V_2^{BD_1^{3/2}}$	-0.09^*	2.14^*	4.21^*
h	0.015	1.67	1.20	k	0.79	1.29	0.93
b_+	-0.013	1.68	0.98	b_-	0.011	1.50	0.91
$F_1^{BD_0^*}$	0.18	0.28	0.25	$F_0^{BD_0^*}$	0.18	–	–
$A^{BD_1^{1/2}}$	-0.16	0.87	0.24	$V_0^{BD_1^{1/2}}$	0.18	0.89	0.25
$V_1^{BD_1^{1/2}}$	-0.19	–	–	$V_2^{BD_1^{1/2}}$	-0.18	0.87	0.24
$A^{BD_1^{3/2}}$	0.16	0.46	0.065	$V_0^{BD_1^{3/2}}$	0.43	0.54	0.074
$V_1^{BD_1^{3/2}}$	0.40	-0.60	1.15	$V_2^{BD_1^{3/2}}$	-0.12	1.45	0.83
h	0.011	0.86	0.23	k	0.60	0.40	0.68
b_+	-0.010	0.86	0.23	b_-	0.010	0.86	0.23

be more prominent in heavy to light transitions, especially at maximum recoil ($q^2 = 0$). For example, we obtain $V_0^{Ba_1}(0) = 0.13$ [13], while ISGW2 gives 1.01. If $a_1(1260)$ behaves as the scalar partner of the ρ meson, it is expected that $V_0^{Ba_1} \sim A_0^{B\rho} \sim O(0.1)$.

5. To determine the physical form factors for $B \to D_1(2427), D_1(2420), D_{s1}(2460), D_{s1}(2536)$ transitions, one needs to know the mixing angles of $D_1^{1/2} - D_1^{3/2}$. A mixing angle $\theta_{D_1} = (5.7 \pm 2.4)^\circ$ is obtained by Belle through a detailed $B \to D^*\pi\pi$ analysis [16], while $\theta_{D_{s1}} \approx 7^\circ$ is determined from the quark potential model [29] as the present upper limits on the widths of $D_{s1}(2460)$ and $D'_{s1}(2536)$ do not provide any constraints on the $D_{s1}^{1/2}$–$D_{s1}^{3/2}$ mixing angle. We use a theoretical predicted $\theta_{D_1} = 12^\circ$ in Table IV [29].

6. The decay rates for $\overline{B} \to D^{**}\pi,\ D^{**}\rho,\ \overline{D}_s^{**}D^{(*)}$ obtained in light-front and ISGW2 models shown in Table IV agree with experimental results in most cases. (i) Note that our decay constants for p-wave mesons are used in both LF and ISGW2 cases. (ii) Usually we expect a factor two to three enhancement in $D^{**}\rho$ rates from $D^{**}\pi$ rates.

TABLE IV: Updated decay rates for $\overline{B} \to D^{**}\pi,\ D^{**}\rho,\ \overline{D}_s^{**}D^{(*)}$ obtained in light-front and ISGW2 models respectively [29]. Since decay constants for p-wave mesons are not provided in ISGW2 model, we use our decay constants and their form factors for the ISGW2 results quoted below.

$\mathcal{B}(10^{-3})$	LF	ISGW2	Expt	Ref
$B^- \to D_0^*(2308)^0\pi^-$	0.83	0.52	0.92 ± 0.29	Belle
$B^- \to D_1(2427)^0\pi^-$	0.52	1.1	0.75 ± 0.17	Belle
$B^- \to D'_1(2420)^0\pi^-$	1.3	1.0	1.0 ± 0.2	Belle,BaBar
			1.5 ± 0.6	PDG
$B^- \to D_2^*(2460)^0\pi^-$	1.2	0.66	0.78 ± 0.14	Belle,BaBar
$B^- \to D_0^*(2308)^0\rho^-$	1.7	1.0		
$B^- \to D_1(2427)^0\rho^-$	1.1	2.1		
$B^- \to D'_1(2420)^0\rho^-$	3.7	2.6	< 1.4	PDG
$B^- \to D_2^*(2460)^0\rho^-$	3.4	1.8	< 4.7	PDG
$B^- \to \overline{D}_{s0}^*(2317)^-D^0$	1.3	–	0.85 ± 0.33	Belle
$B^- \to \overline{D}_{s1}(2460)^-D^0$	1.9	–	$1.5 \sim 4.4$	Belle
$B^- \to \overline{D}'_{s1}(2536)^-D^0$	0.55	–		
$B^- \to \overline{D}_{s0}^*(2317)^-D^{*0}$	0.70	–		
$B^- \to \overline{D}_{s1}(2460)^-D^{*0}$	7.1	–		
$B^- \to \overline{D}'_{s1}(2536)^-D^{*0}$	1.4	–		

The old upper limit in $D'_1(2420)\rho$ needs further check. (iii) The color-allowed Cabbibo favored $\overline{B} \to \overline{D}_s^{**}D^{(*)}$ amplitudes involve $\overline{B} \to D^{(*)}$ form factors and D_s^{**} decay constants. Since the form factors are standard, this type of decays provides valuable information on D_s^{**} decay constants. The agreement between theory and experiment supports our predictions on D_s^{**} decay constants.

IV. ISGUR-WISE FUNCTIONS

In [13] Isgur-Wise functions are obtained through either top down [9] or bottom up (take $m_Q \to \infty$) approaches. The IW functions can be fitted nicely to the form

$$
\begin{aligned}
\xi(\omega) &= 1 - 1.22(\omega - 1) + 0.85(\omega - 1)^2, \\
\tau_{1/2}(\omega) &= 0.31\left(1 - 1.18(\omega - 1) + 0.87(\omega - 1)^2\right), \\
\tau_{3/2}(\omega) &= 0.61\left(1 - 1.73(\omega - 1) + 1.46(\omega - 1)^2\right), \quad (9)
\end{aligned}
$$

where we have used the same β_∞ parameter for both initial and final wave functions. Our results are similar to that obtained in the ISGW model [17] (numerical results for the latter being quoted from [34]). Our result $\rho^2 = 1.22$ for the slope parameter is consistent with the current world average of 1.44 ± 0.14 extracted from exclusive semileptoic B decays [35].

It is interesting to notice that there are Uraltsev and

Bjorken sum rules [19, 20]

$$\sum_n |\tau_{3/2}^{(n)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2 = \frac{1}{4}\,,$$
$$\rho^2 = \frac{1}{4} + \sum_n |\tau_{1/2}^{(n)}(1)|^2 + 2\sum_n |\tau_{3/2}^{(n)}(1)|^2\,, \tag{10}$$

respectively, where n stands for radial excitations and ρ^2 is the slope of the IW function $\xi(\omega)$. The Bjorken and Uraltsev sum rules for the Isgur-Wise functions are fairly satisfied.

V. CONCLUSIONS

We have studied the decay constants and form factors of the ground-state s-wave and low-lying p-wave mesons within a covariant light-front approach. Our main results are as follows: (i) The SU(N) and HQ relations on decay constants are satisfied. (ii) The decay constant of scalar mesons is suppressed relative to that of the pseudoscalar mesons. The smallness of the decay constant of the newly observed $D_{s0}^*(2317)$ implied by a recent Belle measurement on $B \to \overline{D} D_{s0}^*$ could be accommodated. (ii) Numerical results of the form factors for $B \to D,\ D^*,\ D^{**}$ transitions are presented in detail. At $q^2 = 0$ our results are close to ISGW2 results within 40 %. (iii) The prediced decay rates for $\overline{B} \to D^{**}\pi$, $D^{**}\rho$, $\overline{D}_s^{**} D^{(*)}$ obtained in light-front and ISGW2 models shown in Table IV agree with experimental results. (iv) The universal Isgur-Wise functions $\xi(\omega), \tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ are obtained. The Bjorken and Uraltsev sum rules for the Isgur-Wise functions are fairly satisfied.

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